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Saturn-ring defects around microspheres suspended in nematic liquid crystals: An analogy between confined geometries and magnetic fields

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Particles suspended in a nematic liquid crystal exhibit characteristic dipolar and Saturn-ring configurations. Using results on the magnetic-field behavior of these configurations, we explain the recent observation of the Saturn-ring defect in confined geometries based on the idea that a confining geometry and a magnetic field generate a similar "confinement" for the nematic phase.

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The dispersion of colloidal particles in a nematic liquid crystal constitutes a new attractive soft matter system [1–4] since new interesting phenomena arise through the combination of colloidal suspensions and liquid crystals [5–8]. Two configurations exist when single particles with rigid radial anchoring of the liquid crystal molecules at their surfaces are placed in a uniformly aligned nematic phase. In the dipole configuration the particle is accompanied by a topological point defect [1], whereas in the Saturn-ring configuration the particle is surrounded by a $-1/2$ disclination ring at the equator [3]. In recent beautiful experiments, Gu and Abbott reported the observation of the Saturn-ring configuration around glass spheres coated with thin films of gold. Their radii were 20 or 50 μm [9]. At first glance these experiments seem to contradict theoretical findings which demonstrate that in infinite systems the dipolar configuration should always be the stable configuration for particle radii larger than approximately 300 nm and that the Saturn-ring configuration becomes absolutely unstable for radii larger than 700 nm (all, of course, under the assumption of rigid surface anchoring of the molecules) [10]. The main purpose of this Brief Report is to resolve this contradiction. We will argue that the observation of the Saturn-ring configuration is due to the fact that the particles are not situated in an infinitely extended volume but that they are placed in a confined geometry given by the liquid crystal (LC) cell which holds the nematic colloidal suspension. The second reason for this Brief Report is to express the idea that both a confining geometry and a magnetic field generate a similar "confinement" for the nematic phase that leads to equivalent responses, which in our case is the stabilization of the Saturn ring configuration. Another example is director fluctuations which become massive either by applying a magnetic field or by confining the liquid crystal to a finite volume [11]. In our reasoning for the observation of the Saturn ring in the experiment mentioned above, we will therefore employ results of the magnetic-field behavior of nematic colloids [10]. In a more general context, it is important that a confining geometry and a magnetic field constitute an external control for the nematic colloidal system [12] to which it can respond in a similar manner.

In the experiments by Gu and Abbott, the microsphere was situated in the middle between two bounding parallel plates of the LC cell which supplied a planar anchoring for the LC molecules. In the following we assume rigid anchoring at both the particle's surface and the bounding plates. This is reasonable since with the averaged Frank elastic constant $K=10^{-6}$ dyn and a high anchoring strength of, e.g., $W=10^{-1}$ erg/cm^2 [13], we arrive at a surface extrapolation length $\xi_S=K/W=0.1$ μm much smaller than the radius of the microsphere and also the distance Δd between the particle's surface and the bounding plates. In addition, the occurrence of the Saturn ring also suggests $\xi_S \ll a$; otherwise the surface ring configuration should be observed [10,14]. Thus the confinement by the LC cell forces a uniform alignment close to the microsphere which, according to our assertion, stabilizes the Saturn-ring configuration. At this point we introduce our idea that a magnetic field $\boldsymbol{H}$ pointing along the symmetry axis of the Saturn-ring configuration produces a similar alignment or "confinement" where the magnetic coherence length ξ_H replaces the distance Δd of the microsphere from the bounding plates. The length $\xi_H = [K/(\Delta\chi H^2)]^{1/2}$ (where $\Delta\chi$ is the magnetic anisotropy) denotes the distance which the magnetic field needs to align the nematic director parallel to its direction [15]. The only difference compared to the two bounding surfaces of the LC cell is that the magnetic field produces a confinement with cylindrical symmetry. However, we are only interested in the main mechanism and therefore can safely disregard this difference. To work out its consequences, a detailed numerical study of both geometries is necessary. A simple argument demonstrates that for $\xi_H \ll a$ (a is the particle radius) the Saturn ring is the preferred configuration [16]. For $\xi_H \ll a$, Fig. 1 sketches the director field of the dipole (left) and the Saturn-ring (right) configuration. The director field is basically aligned along the magnetic field. In the dipolar case, strong distortions only occur in a layer of thickness ξ_H

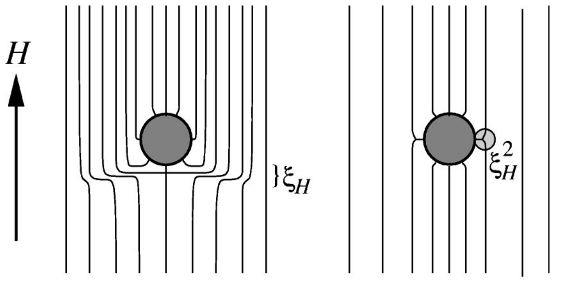

FIG. 1. Sketch of the director field lines in the case of $\xi_H \ll a$ for the dipole (left) and the Saturn-ring (right) configuration.

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and a volume which scales like $\xi_H a^2$. In the case of the Saturn ring, the volume of strong distortions is given by a torus of cross section $\propto \xi_H^2$ enclosing the ring disclination, hence it scales as $\xi_H^2 a$. Since both the Frank free-energy density in the one-constant approximation $[K(\nabla_i n_j)^2/2]$ and the magnetic free-energy density relative to the aligned director field $(-\Delta\chi[(n_i H_i)^2 - H^2]/2)$ scale like $1/\xi_H^2$, the total free energy of the Saturn ring is smaller by a factor of a/ξ_H compared to the dipole. We expect a transition between both configurations to occur at $\xi_H \propto a$. This is confirmed by numerical calculations for $a = 0.5$ μm where the transition takes place at $a/\xi_H \approx 0.33$ [10]. In recent experiments, the transition was observed by Loudet and Poulin with the help of electrical fields [17] where our reasoning applies as well when ξ_H is replaced by the equivalent electric coherence length ξ_E [15]. We also note that a field-induced transition from a point to a ring defect has already been addressed for the reverse case of a nematic droplet in an isotropic matrix in Ref. [18]. Let us return to the experiments by Gu and Abbott. Their LC cell had a thickness of 120 μm and the particles were situated close to the center of the LC cell. Replacing ξ_H by Δd, as explained above, we find for the large particles ($a = 50$ μm) a ratio $a/\Delta d = 5$, for which we expect the Saturn ring to occur according to our analogy. For the small particles ($a = 20$ μm), $a/\Delta d = 0.5$, which should be close to the transition point.

In conclusion, by using an analogy with an applied magnetic field, we confirm the finding of Gu and Abbott who observed Saturn ring configurations around large particles in a confined geometry. It would be interesting to study the transition to the dipole configuration in more detail by varying $a/\Delta d$. Metastability regions for both configurations are expected in analogy to an applied magnetic field [10]. Interestingly, spontaneous transitions from the Saturn ring to a dipolar object with a complex defect structure were already reported by Gu and Abbott; however, they did not specify the ratio $a/\Delta d$ [9]. For a comparison with more detailed experiments, a numerical study is necessary.

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[1] P. Poulin, H. Stark, T.C. Lubensky, and D.A. Weitz, Science **275**, 1770 (1997).

[2] P. Poulin, V.A. Raghunathan, P. Richetti, and D. Roux, J. Phys. II **4**, 1557 (1994).

[3] E.M. Terentjev, Phys. Rev. E **51**, 1330 (1995).

[4] H. Stark, Phys. Rep. **351**, 387 (2001).

[5] J.-C. Loudet, P. Barois, and P. Poulin, Nature (London) **407**, 611 (2000).

[6] S.P. Meeker, W.C.K. Poon, J. Crain, and E.M. Terentjev, Phys. Rev. E **61**, R6083 (2000).

[7] V.G. Nazarenko, A.B. Nych, and B.I. Lev, Phys. Rev. Lett. **87**, 075504 (2001).

[8] H. Stark and D. Ventzki, Europhys. Lett. **57**, 60 (2002).

[9] Y. Gu and N.L. Abbott, Phys. Rev. Lett. **85**, 4719 (2000).

[10] H. Stark, Eur. Phys. J. B **10**, 311 (1999).

[11] A. Mertelj and M. Čopič, Phys. Rev. E **61**, 1622 (2000).

[12] H. Löwen, J. Phys.: Condens. Matter **13**, R415 (2001).

[13] L.M. Blinov, A.Y. Kabayenkov, and A.A. Sonin, Liq. Cryst. **5**, 645 (1989).

[14] R.W. Ruhwandl and E.M. Terentjev, Phys. Rev. E **56**, 5561 (1997).

[15] P.G. de Gennes and J. Prost, *The Physics of Liquid Crystals*, 2nd ed. (Oxford Science Publications, Oxford, 1993).

[16] In the simple argument, we also assume $\xi_S \ll \xi_H$ to comply with the rigid boundary condition. Given the estimate for ξ_S and the particle radii in the experiment, this assumption is reasonable.

[17] J.C. Loudet and P. Poulin, Phys. Rev. Lett. **87**, 165503 (2001).

[18] V.G. Bodnar, O.D. Lavrentovich, and V.M. Pergamenshchik, Sov. Phys. JETP **74**, 60 (1992) [Zh. Éksp. Teor. Fiz. **101**, 111 (1992)].